ME 243

Mechanics of Solids

Lecture 8: Curved Beams and reinforced beams

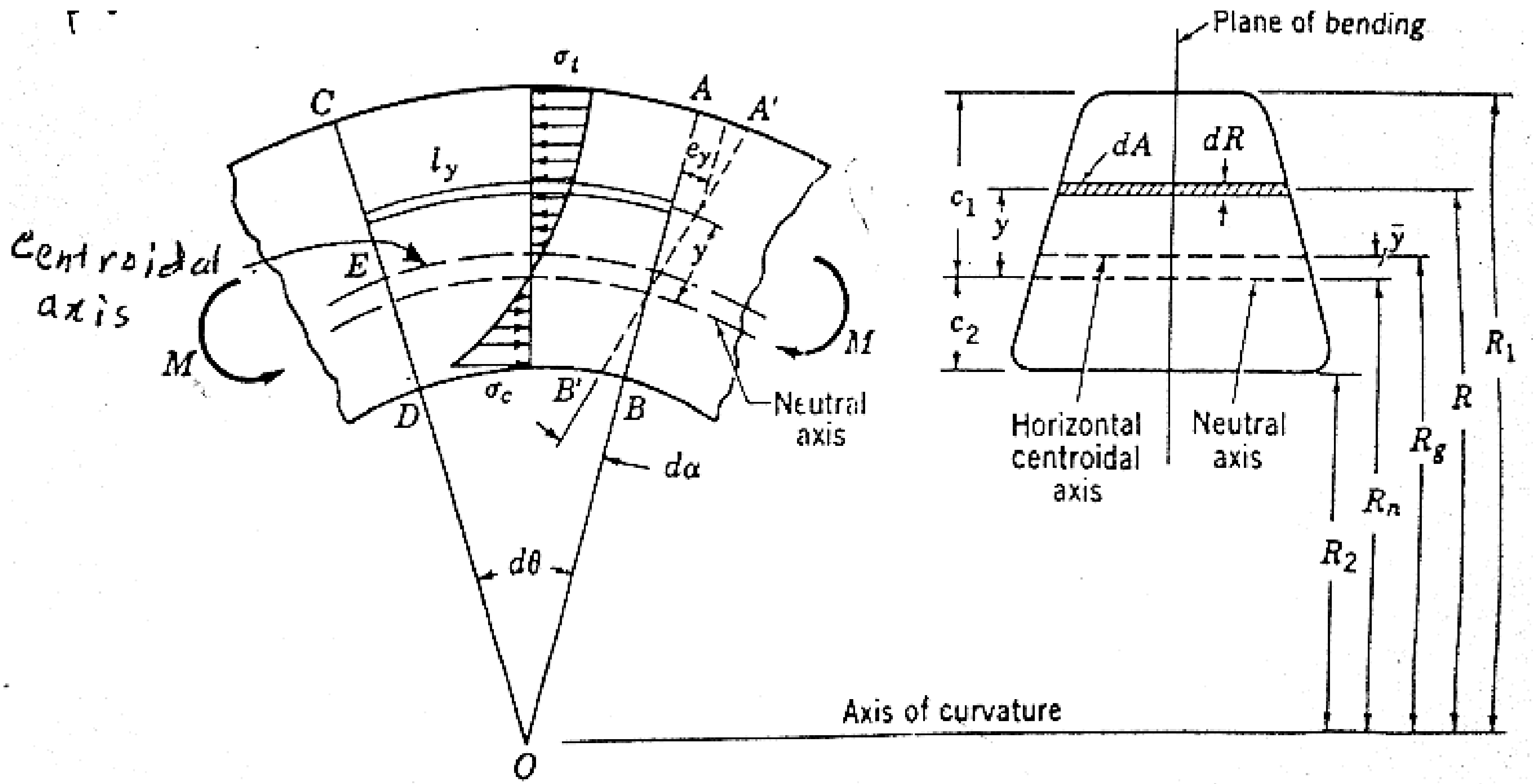
Ahmad Shahedi Shakil

Lecturer, Dept. of Mechanical Engg, BUET

E-mail: sshakil@me.buet.ac.bd, shakil6791@gmail.com

Website: teacher.buet.ac.bd/sshakil

Curved beam theory



Curved beam theory

Unit strain for any fiber y distance from neutral axis is

$$\epsilon_y = \frac{e_y}{l_y} = \frac{y\, d\alpha}{(R_n + y)\, d\theta}$$

The stress on the fiber is

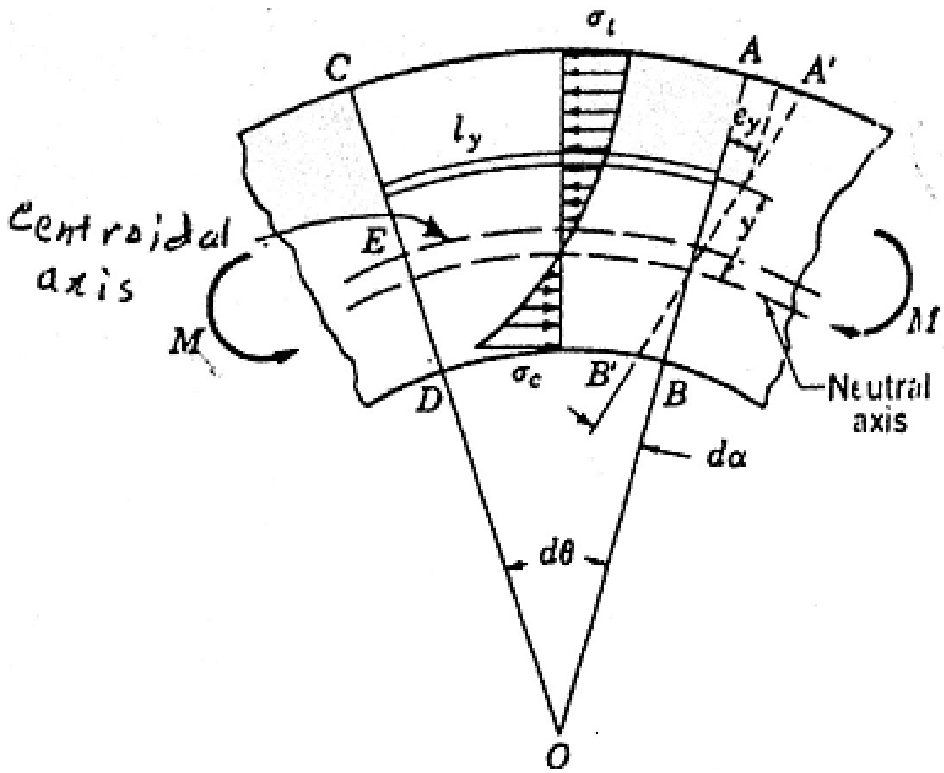
$$\sigma_y = E\epsilon_y = \frac{E\, d\alpha}{d\theta} \times \frac{y}{R_n + y}$$

Since the normal force on any section must be zero for equilibrium, we can write,

$$\int_{R_2}^{R_1} \sigma_y\, dA = \frac{E\, d\alpha}{d\theta} \int_{R_2}^{R_1} \frac{y}{R_n + y}\, dA = 0$$

As the term $E\, d\alpha / d\theta$ is constant , we get,

$$\int_{R_2}^{R_1} \frac{y}{R_n + y}\, dA = 0$$



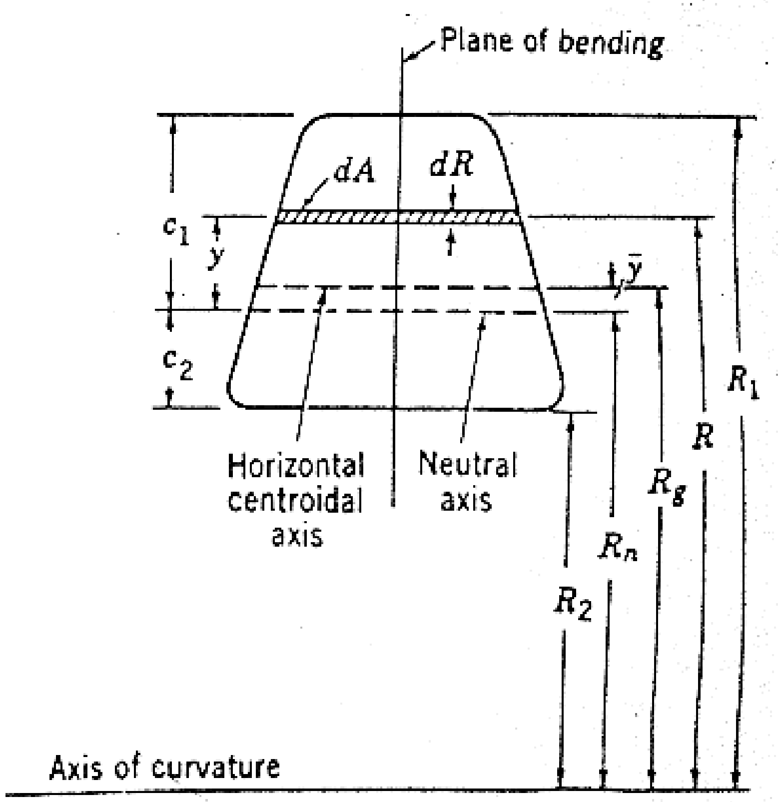
Curved beam theory

Placing $y = R - R_n$, we obtain,

$$\int_{R_2}^{R_1} \frac{R - R_n}{R} dA = 0,$$

Hence, $R_n = \dfrac{A}{\int_{R_2}^{R_1} \frac{dA}{R}}$

$$\bar{y} = R_g - \frac{A}{\int_{R_2}^{R_1} \frac{dA}{R}}$$



So, in bending of a curved beam, the neutral axis shifts away from the centroidal axis and towards the axis of curvature of the beam.

Curved beam theory

The resisting moment of the normal forces acting on the section (C-T couple) must be equal to applied moment. Thus we get,

$$\int_{R_2}^{R_1} y\sigma_y \, dA = \frac{F\,d\alpha}{d\theta}\int_{R_2}^{R_1}\frac{y^2\,dA}{R_n + y} = M \quad \text{..................(1)}$$

Now,

$$\int_{R_2}^{R_1}\frac{y^2\,dA}{R_n + y} = \int_{R_2}^{R_1}\left[y - R_n\left(\frac{y}{R_n + y}\right)\right]dA = \int_{R_2}^{R_1} y\,dA - R_n\int_{R_2}^{R_1}\frac{y}{R_n + y}\,dA$$

Here, the second integral is shown as zero, so,

$$\int_{R_2}^{R_1}\frac{y^2\,dA}{R_n + y} = \int_{R_2}^{R_1} y\,dA = \bar{y}A$$

Curved beam theory

From equation (1), we get,

$$\frac{M}{\bar{y}A} = \frac{F\,d\alpha}{d\theta}$$

So, we can write,

$$\sigma_y = \frac{M}{\bar{y}A} \times \frac{y}{R_n + y}$$

This is a general formula for flexural stress in a curved beam.

For outer fiber, $y=c_1$ and for inner fiber, $y=c_2$.

So we get,

Outer fiber:

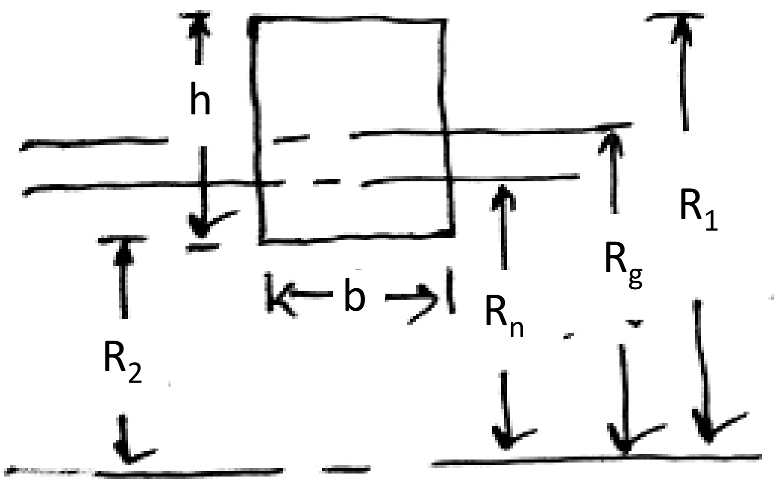
$$\sigma_1 = \frac{Mc_1}{\bar{y}AR_1}$$

Inner fiber:

$$\sigma_2 = \frac{Mc_2}{\bar{y}AR_2}$$

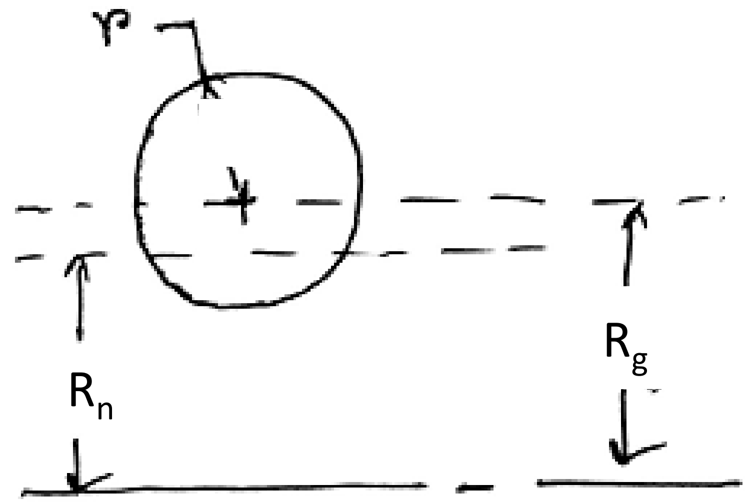
Curved beam theory

Rectangle



$$R_n = \frac{h}{\ln \frac{R_1}{R_2}}$$

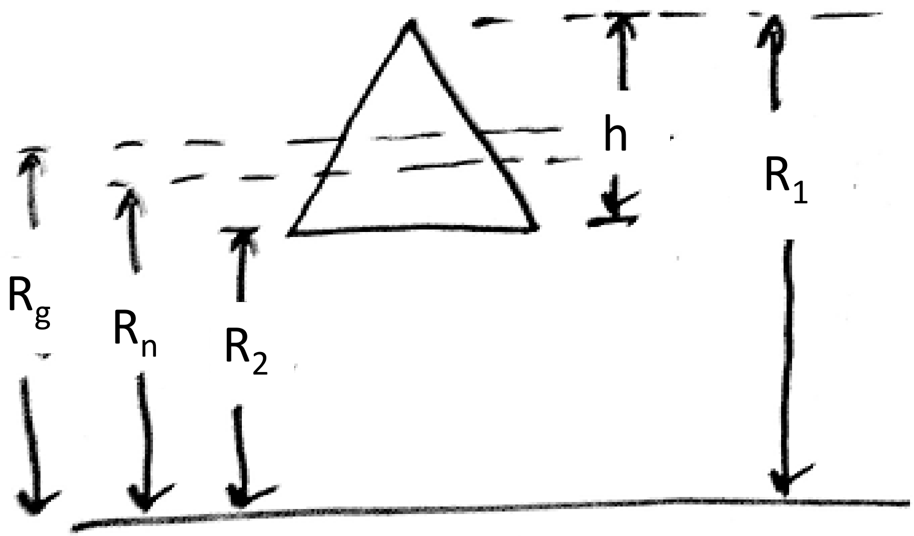
Circle



$$R_n = \frac{1}{2}\left(R_g + \sqrt{R_g^2 - r^2}\right)$$

Curved beam theory

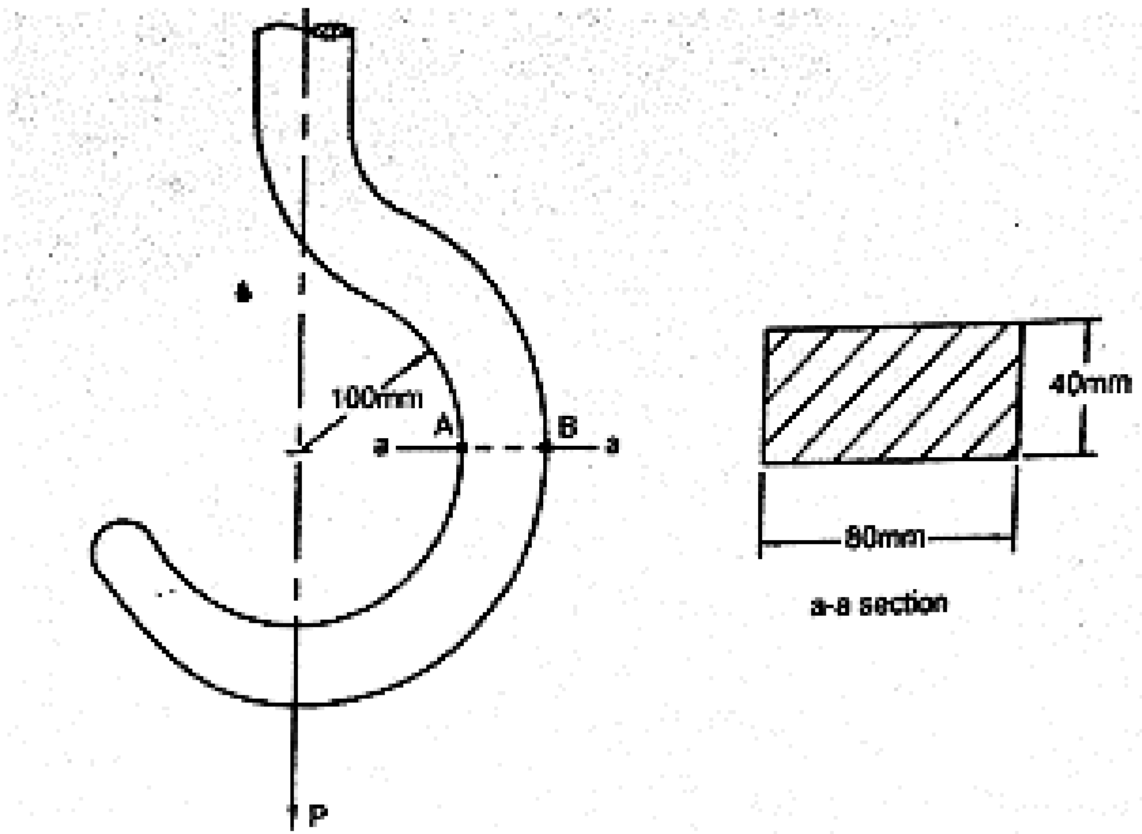
Triangle



$$R_n = \frac{\frac{h}{2}}{\frac{R_1}{h} \ln \frac{R_1}{R_2} - 1}$$

Problem#1018(quamrul)

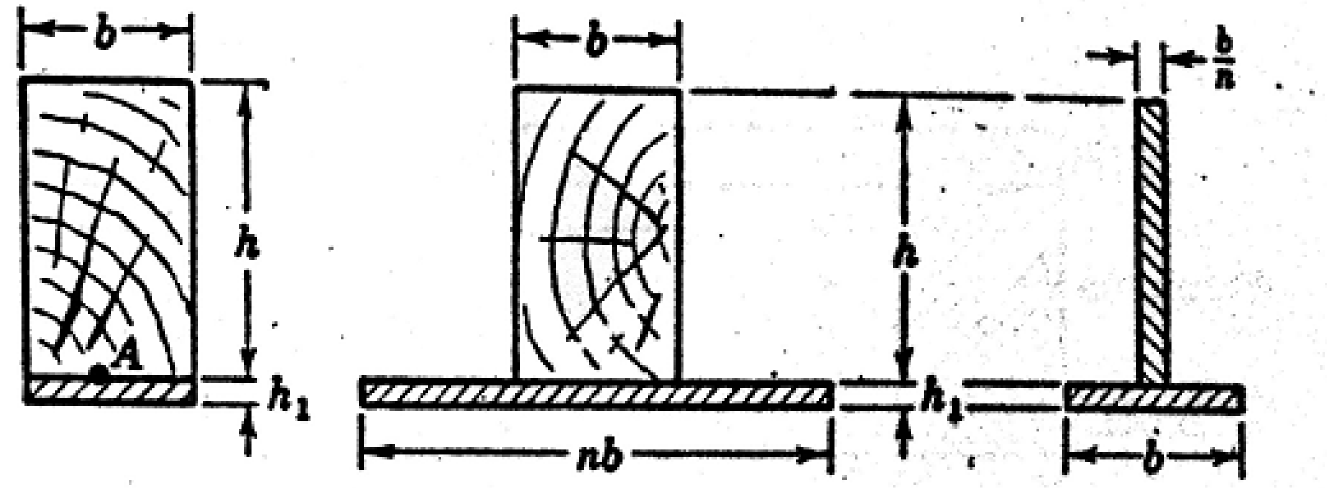
- A crane hook of rectangular cross-section supports a load P= 40 kN as shown in fig. Determine the normal stresses at the points A and B.



Reinforced beam

- It was once common to strengthen timber beams by bolting strips of steel to them.
- The most common type of reinforced beam used today is the concrete beam reinforced with steel rods.
- The theory of flexure does not apply to composite beams because the beam becomes nonhomogeneous.
- The most common method of dealing with a non homogeneous beam is to transform it into an equivalent homogeneous beam to which flexure formula may be applied.

Beams of different materials



(a) Timber and steel section (b) Equivalent wood section (c) Equivalent steel section

The timber beam in figure (a) is fastened with a steel strip for reinforcement. So no slip occurs between them as the beam is bent.

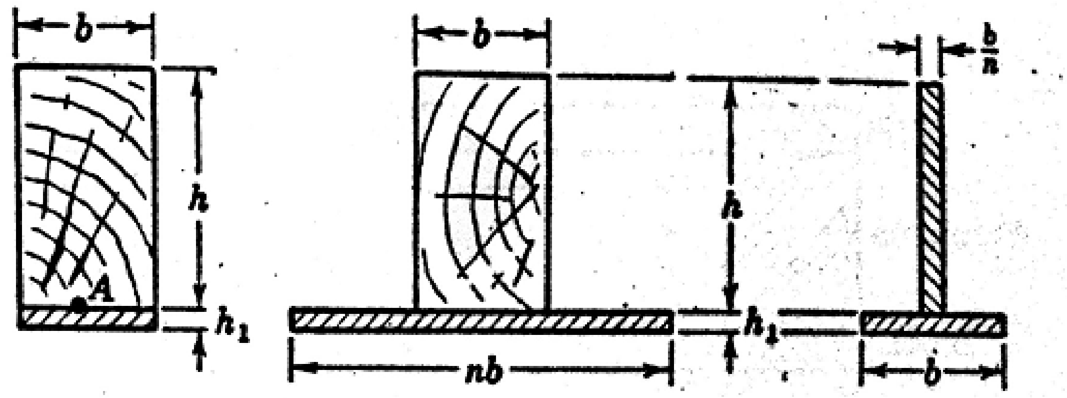
To obtain and equivalent section, consider a longitudinal steel fiber of the beam at A. The strains of the steel and wood fibers at junction A must be equal.

i.e. $\epsilon_s = \epsilon_w$ or, $\frac{\sigma_s}{E_s} = \frac{\sigma_w}{E_w}$

Beams of different materials

The loads carried by the steel fiber and the equivalent wood fiber must be equal. So ,

$$P_s = P_w$$



(a) Timber and steel section (b) Equivalent wood section (c) Equivalent steel section

Or, $A_s \sigma_s = A_w \sigma_w$

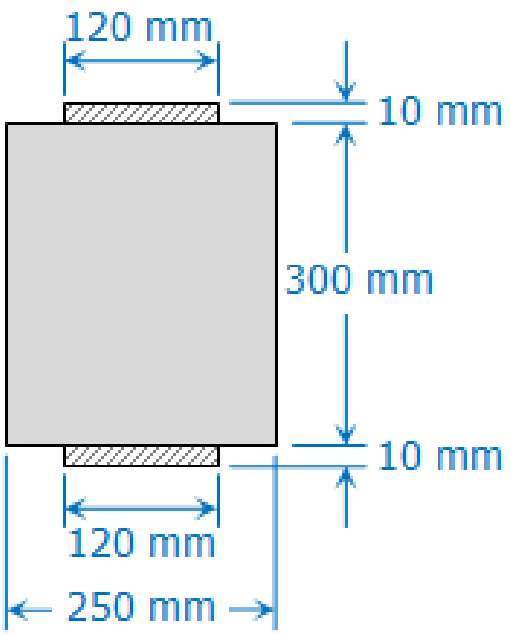
Or, $A_s \left(\frac{E_s}{E_w} \right) \sigma_w = A_w \sigma_w$

Denoting E_s/E_w =n, we can write, $\boxed{A_w = nA_s}$

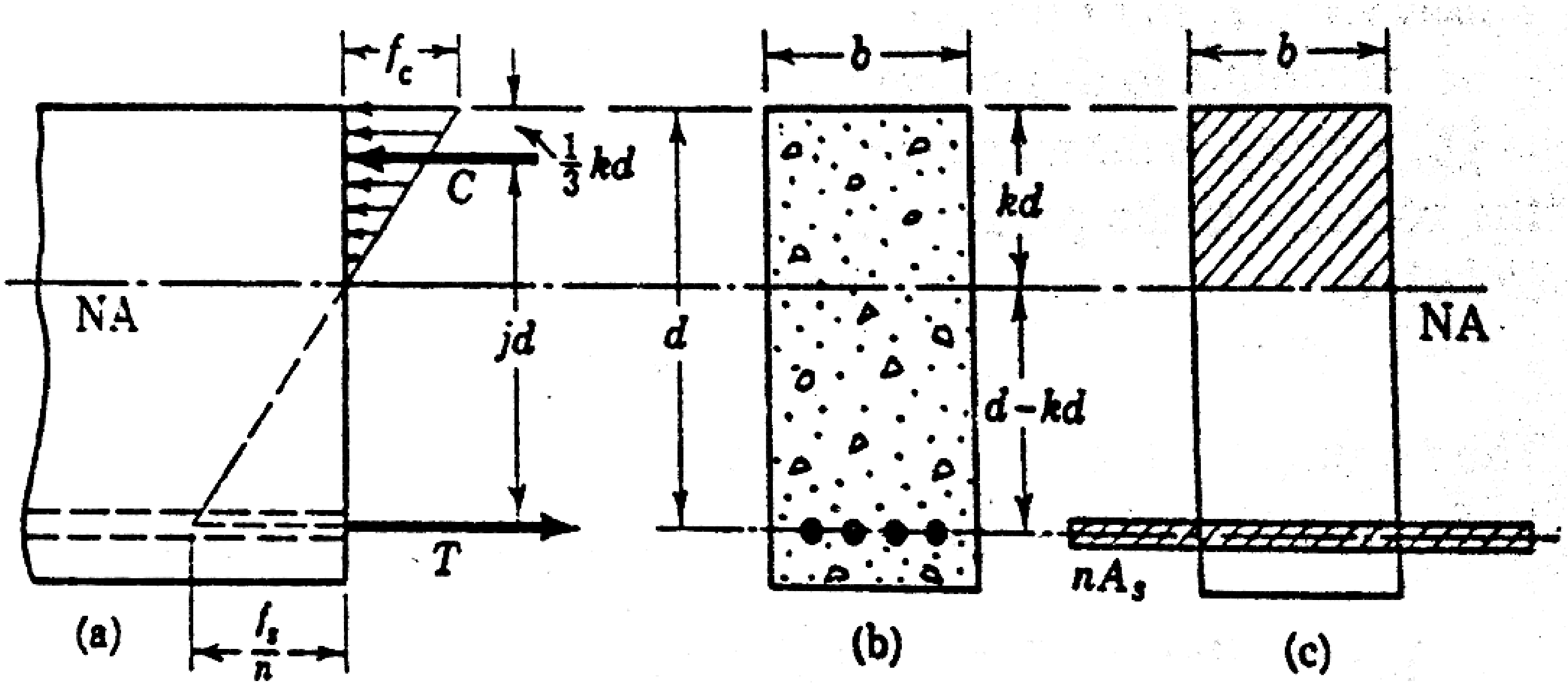
This indicates that the area of the equivalent wood is n times greater than the area of the steel.

Problem#1003 (singer)

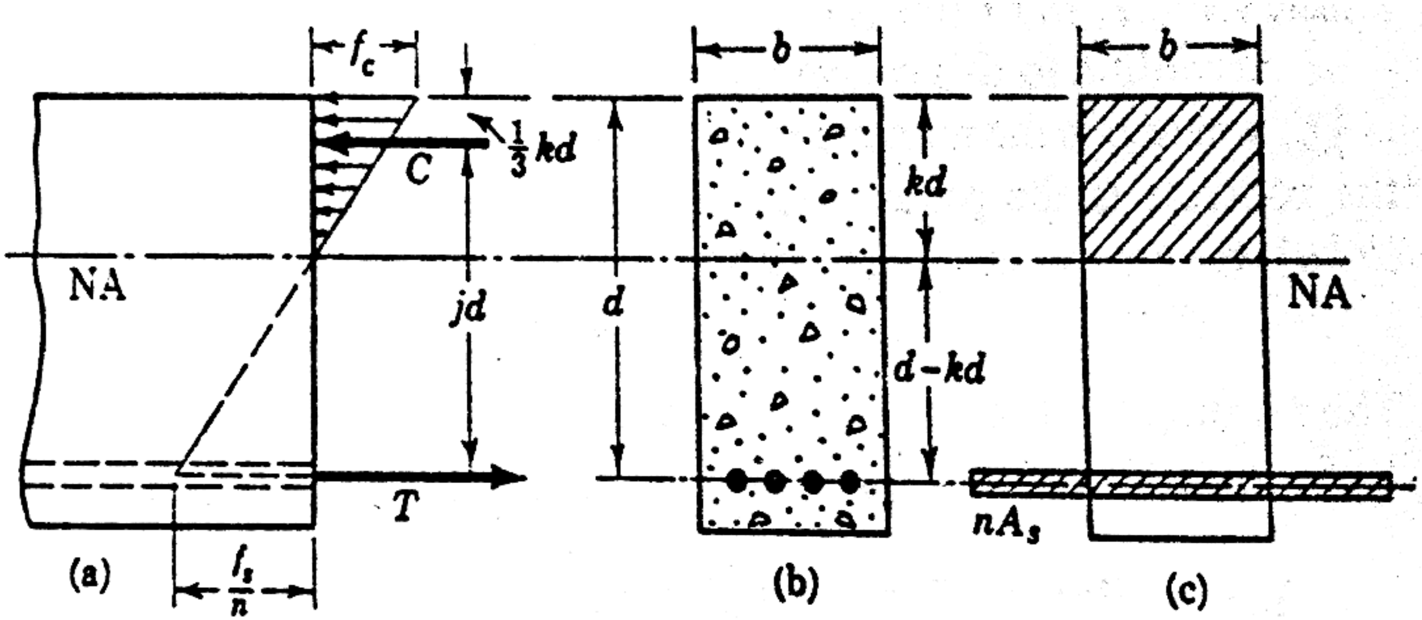
- A simply supported beam 4 m long has the cross section shown in Fig. P-1002. It carries a uniformly distributed load of 20 kN/m over the middle half of the span. If n = 15, compute the maximum stresses in the wood and the steel.



Reinforced concrete beam



- Concrete is an excellent building material because it is cheap and fireproof and does not rust or rot. Its compressive strength is sufficient but its tensile strength is practically zero. For this reason, the tensile side of the concrete beam is reinforced with steel bars.



The portion of the reinforced concrete beam in fig. (a) has the cross section shown in fig. (b). The equivalent section in terms of concrete is shown in fig (c).

Here, $n = E_s/E_c$.

If the quantities b, d, A_s and n are known, the neutral axis is obtained by the principle that the moment of area above the neutral axis equals the moment of area below the axis. i.e.,

$$b \cdot kd\left(\frac{kd}{2}\right) = nA_s(d - kd)$$

- The resultant compressive force C acts at a distance (1/3)kd from the top fiber. The resulting couple, composed of the equal compressive and tensile forces C and T has a moment arm of jd.

$$jd = d - (1/3)kd$$

Let, f_c= maximum compressive stress in concrete

f_s= tensile stress in steel.

The average compressive stress in the concrete is $(1/2)f_c$. So,

$$C = (1/2)\, f_c\, (b.\, kd)$$

The resisting moment based on compressive stress is,

$$\boxed{M_c = C(jd) = (1/2)\, f_c\, (b.\, kd)\, (jd)}$$

The resisting moment in terms of steel is,

$$\boxed{M_s = T(jd) = f_s A_s (jd)}$$

Problem# 1027 (singer)

- Determine the maximum stresses developed in the concrete and steel of a reinforced beam by a bending moment of 70 kN.m if b=300 mm, d= 500 mm, As= 1200 mm and n=8